PMM U.S.S.R.,Vol.54,No.1,pp.19-25,1990
Printed in Great Britain

0021-8928/90 $10.00+0.00

STABILITY OF A HORIZONTAL ROTOR IN FLEXIBLE ROLLING BEARINGS WITH CLEARANCES*

A.S. KEL'ZON and A.A. KOVAL

The stability of motion of a horizontal rotor in flexible rolling bearings with radial clearances, treated as a system with distributed mass, is analysed. An algorithm is derived for calculating the equations of the boundary curves of the stable and unstable regions, by determining the zeros of the determinants of certain infinite block matrices. The vertices of the parametric resonance zones that may occur in this non-linear system are determined in explicit form. It is shown that the parametric resonances may be found by determining the natural frequencies of the corresponding linear systems with reduced values of the stiffnesses of the bearings with clearances.

The possibility that clearances in the bearings may significantly affect the dynamics of a system was first pointed out in /1, 2/. A detailed analysis is available /3/ of the motion of a shaft in bearings with clearances.

Previous studies of rotor stability /1-3/ have considered the rotor as a point mass revolving in a flexible bearing with clearance. This scheme, however, is highly idealized and does not make allowance for many features of real rotors as elastic systems with distributed mass. The problem is of particular importance for high-rpm rotors revolving at an angular velocity above the first or second critical velocity of the system.

1. The problem of the forced oscillations of a rotor as a system with distributed mass can be solved using the method of initial parameters /4/. To do this, the real rotor is divided into a series of stepped segments of constant linear mass and flexural rigidity, in such a way that different machine parts mounted on the shift (discs, concentrated masses, unbalances, and bearings)are situated at the ends of the appropriate segments (Fig.1).

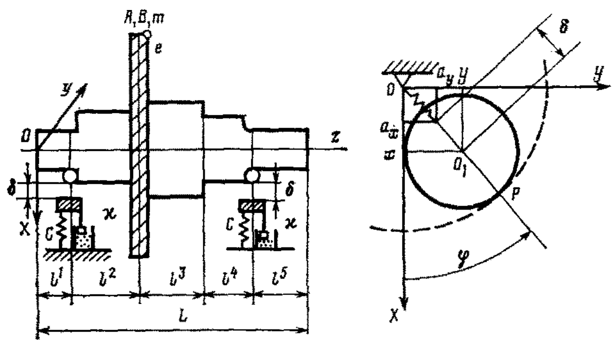

Fig.1

We introduce a right-handed coordinate system $OXYZ$ attached to the stationary rotor: O is the centre of the left cross-section of the rotor, the Z-axis points along the rotor axis, the X-axis is vertically upward and the Y-axis is horizontal (Fig.1). Also shown in the figure is a cross-section of the rotor in the plane of the bearing: O_1 is the centre of the rotor journal, x, y are the projections of the journal displacement on the X, Y axes, φ is the angle of precession, and a_x, a_y are the projections of the deformation vector on the X, Y axes. The dotted line indicates the surface of the bearing.

The equations of the variations for the k-th homogeneous segment of the rotor are /4/

$$EJ^k u^k_{zzzz} + \mu^k u^k_{tt} + \mu^k g = 0 \tag{1.1}$$

where EJ^k is the flexural stiffness of the segment, μ^k is its mass per unit length, $u^k(z, t)$, and $v^k(z, t)$ are the projections of the rotor displacement on the X and Y axes, respectively (the indices z and t indicate differentiation with respect to the coordinate z and the time

*Prikl.Matem.Mekhan.,54,1,26-33,1990

t).

The matching equations, relating the vibration parameters at the end of the k-th and the beginning of the $(k+1)$-th segments of the rotor, in the case of uninterrupted vibrations (the first and third types of motion), are as follows /5/ (omitting the argument t in the functions u and v):

$$\begin{gathered} u^k(l^k) = u^{k+1}(0), \quad u_z^k(l^k) = u_z^{k+1}(0) \\ EJ^k u_{zz}^k(l^k) = EJ^{k+1} u_{zz}^{k+1}(0) + B^k u_{ttz}^{k+1}(0) + A^k \omega v_{tz}^{k+1}(0) \\ EJ_{zzz}^k(l^k) = EJ^{k+1} u_{zzz}^{k+1}(0) + C^k a_x^k + m^k u_{tt}^{k+1}(0) + \varkappa^k v^{k+1}(0) - m^k g - \\ m^k e^k \omega^2 \cos(\omega t + \gamma^k) \end{gathered} \tag{1.2}$$

(the matching equations for v are similar). Here l^k is the length of the k-th segment, B^k and A^k are the equatorial and polar moments of inertia of a disc mounted at the end of the k-th segment, ω is the frequency of revolution of the rotor, C^k and $\varkappa^k$ are the stiffness and damping of the bearing at the end of the k-th segment, m^k is the concentrated mass; e^k is the eccentricity of the mass m^k, $m^k e^k \omega^2$ is the centrifugal force due to the eccentricity e^k, and γ^k is the angle between the eccentricity and the X axis at the initial time $t=0$.

The equations of motion (1.1), the matching conditions (1.2) and the boundary conditions at the rotor ends ($z=0$ and $z=L$), which are, say, for a rotor with free ends,

$$z = 0, L, \ u_{zz} = u_{zzz} = v_{zz} = v_{zzz} = 0 \tag{1.3}$$

completely define the solution of the problem of forced flexural vibrations of the rotor.

Note that in the case of linear bearings with radial clearance $\delta = 0$, the deformations of the bearing are identical with the displacements of the rotor journal: $a_x^k = u^k$, $a_y^k = v^k$.

The solution of the linear problem will be sought in the form

$$u^k = R_1^k \cos \omega t + R_2^k \sin \omega t, \quad v^k = R_3^k \cos \omega t + R_4^k \sin \omega t \tag{1.4}$$

Eqs.(1.1)-(1.3) yield a matrix equation relating the values of the vibration parameters at the beginning of two consecutive rotor segments:

$$\begin{gathered} \Pi \times (\mathbf{R}_1^k, \mathbf{R}_2^k, \mathbf{R}_3^k, \mathbf{R}_4^k)^T = (\mathbf{R}_1^{k+1}, \mathbf{R}_2^{k+1}, \mathbf{R}_3^{k+1}, \mathbf{R}_4^{k+1})^T \\ \mathbf{R}_i^k = (R_i^k(0), R_{iz}^k(0), R_{izz}^k(0), R_{izzz}^k(0)), \quad i = 1, 2, 3, 4 \end{gathered} \tag{1.5}$$

(Π is a square 16x16 matrix).

Writing Eqs.(1.5) in sequence for all rotor segments and using the rotor end conditions (1.3), we obtain

$$\begin{gathered} AA(\omega, EJ^k, B^k, A^k, C^k, m^k, \varkappa^k)(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4)^T = \\ BB(\omega, m^k, e^k, \gamma^k), \quad \mathbf{p}_i = (R_i(0), R_{iz}(0), R_i(L), R_{iz}(L)) \end{gathered} \tag{1.6}$$

(AA is an 8x8 matrix and BB is an eight-dimensional vector).

2. The non-linearity of the problem lies in the non-linear reaction forces of bearings with clearances. Taking into account that in a rolling bearing the point of contact, the centre of the cross-section of the revolving rotor O_1 and the centre of the undeformed bearing O (Fig.1) lie in the same straight line, we can express the projections a_x and a_y of the deformation vector as follows (omitting the index k for brevity):

$$\begin{gathered} a_x = u - \delta \cos \varphi = u\,(1 - \delta/(u^2 + v^2)^{1/2}) \\ a_y = v - \delta \sin \varphi = v\,(1 - \delta/(u^2 + v^2)^{1/2}) \end{gathered} \tag{2.1}$$

where δ is the radial clearance in the bearing (Fig.1).

We shall consider small pendular vibrations of the shaft in the bearing. In that case the constant component of the vibrations of the shaft journal is approximately equal to the full static sag of the shaft, with allowance for the clearance, and is much larger than the other components of the vibrations:

$$u = l_0 + x = r_0 + \delta + x, \ v = y; \quad l_0 \gg |x|, \ |y| \tag{2.2}$$

(r_0 is the static sag of the shaft in the bearing).

For small pendular vibrations the angle φ is small, and so

$$\sin \varphi \approx y \mid l_0, \quad \cos \varphi \approx 1 - y^2/(2l_0^2) \tag{2.3}$$

Using conditions (2.2), let us expand (2.1) in powers of the small quantities x/l_0, y/l_0. Substituting the results into formulae (1.2) and dropping terms of higher order than $(x/l_0)^3$ and $(y/l_0)^3$, we obtain the conditions for the jump in the transverse force across a section with a bearing:

$$F_x = -Cx + mx_{tt} + \varkappa x_t - C\delta y/(2l_0^2) + C\delta y^2 x/l_0^3 \qquad (2.4)$$
$$F_y = -C(1 - \delta/l_0)y + my_{tt} + \varkappa y_t + C\delta xy/l_0^2 + C\delta x^2 y|\,l_0^3$$

In the first approximation, the solution of the problem of the forced pendular vibrations of the shaft may be assumed to have the form (1.4), with the coefficients R_i^k determined by solving the appropriate linear problem for a shaft in bearings without clearance with twice the stiffness of the original shaft in the horizontal and vertical planes:

$$C_x = C, \quad C_y = C_M = C(1 - \delta/l_0) \qquad (2.5)$$

Then we obtain

$$F_x = -C_x + mx_{tt} + \varkappa x_t - x(b_1 \cos 2\omega t + c_1 \sin 2\omega t) \qquad (2.6)$$
$$F_y = -C_M y + my_{tt} + \varkappa y_t - y(b_2 \cos \omega t + c_2 \sin \omega t)$$
$$b_1 = \frac{R_3^2}{2l_0^2}\Delta, \quad c_1 = \frac{R_3R_4}{2l_0^2}\Delta, \quad b_2 = \frac{R_1}{l_0}\Delta, \quad c_2 = \frac{R_2}{l_0}\Delta, \quad \Delta = \frac{C\delta}{l_0}$$

3. If the effect of the concentrated torques due to discs and the concentrated forces defined by formulae (2.4) is included for some segments of the rotor at $z = z^k$, the equations of motion may be written in the form

$$(EJ(z)x_{zz})_{zz} - \sum_k ((B^k x_{ztt} - A^k \omega y_{zt})h(z - z^k))_z + \qquad (3.1)$$
$$\mu(z)x_{tt} + \sum_k (f_x^k(t)x + m^k x_{tt} + \varkappa^k x_t)h(z - z^k) = 0$$

The equations for the function $y(t, z)$ are similar in form, with

$$f_x^k(t) = C^k + b_1^k \cos 2\omega t + c_1^k \sin 2\omega t \qquad (3.2)$$
$$f_y^k(t) = C^k(1 - \delta^k|\,l_0^k) + b_2^k \cos \omega t + c_2^k \sin \omega t$$

Summation over k runs from 1 to M, where M is the total number of all discs and bearings along the shaft, and $h(z)$ is a delta-function.

We consider the solution as an expansion in terms of eigenfunctions (summation over i runs from 1 to N, integration with respect to z from zero to L):

$$x = \sum_i x_i(z)\mathbf{q}_i(t), \quad y = \sum_i y_i(z)g_i(t) \qquad (3.3)$$
$$\int x_i(z)x_j(z)\mu(z)\,dz = \Delta_{ij}$$
$$\int (EJ(z)x_{izz}(z))_{zz}x_j(z)\,dz = p_i^2\Delta_{ij} - \sum_k C^k x_i^k x_j^k$$

Here Δ_{ij} is the Kronecker delta, p_i are the natural frequencies of the linear system when all $\delta^k = 0$ (the analogous relations for the functions $y_i(z)$ are not given).

Substituting series (3.3) into Eq.(3.1) and the analogous equations for $y(t, z)$, and then multiplying these equations by $x_j(z)$ and $y_j(z)$, respectively, and integrating from zero to L, we obtain

$$p_j^2\mathbf{q}_j(t) + \mathbf{q}_{jtt}(t) - \sum_k\sum_i \int ((B^k x_{iz}\mathbf{q}_{itt}x_j - A^k\omega y_{iz}g_{it}x_j)h(z - z^k))_z\,dz + \qquad (3.4)$$
$$\sum_k\sum_i (f_x^k(t)\mathbf{q}_i x_i^k x_j^k + m^k\mathbf{q}_{itt}x_i^k x_j^k + \varkappa^k\mathbf{q}_{it}x_i^k x_j^k = 0$$

and similar equations, obtained by making the following substitution in (3.4):

$$\mathbf{q}_j(t) \leftrightarrow g_j(t), \quad x_i \leftrightarrow y_i, \quad f_x^k(t) \leftrightarrow f_y^k(t), \quad A^k \leftrightarrow -A^k.$$

Since /6/

$$\int (B^k x_{iz}x_j h(z - z^k))_z\,dz = -\int B^k x_{iz}x_{jz}h(z - z^k)\,dz$$

and similar relations hold for the other terms representing the effect of discs in (3.4), it follows that the above systems of equations can be written in matrix notation:

$$\begin{gathered} M\mathbf{Q}_{tt} + P_x(t)\mathbf{Q} - A\mathbf{G}_t + S\mathbf{Q}_t = 0 \\ M\mathbf{G}_{tt} + P_y(t)\mathbf{G} + A\mathbf{Q}_t + S\mathbf{G}_t = 0 \\ \mathbf{Q} = \mathbf{Q}(t) = (q_1, \ldots, q_N)^T, \quad \mathbf{G} = \mathbf{G}(t) = (g_1, \ldots, g_N)^T \end{gathered} \tag{3.5}$$

where M, P_x, P_y, A, S are symmetric $N \times N$ matrices.

Equating the damping in the bearings to zero (i.e., $S = 0$), we obtain from (3.5)

$$\begin{gathered} L \times \mathbf{R}_{tt} + D \times \mathbf{R}_t + P(t) \times \mathbf{R} = 0 \\ \mathbf{R} = \begin{Vmatrix} \mathbf{Q} \\ \mathbf{G} \end{Vmatrix}, \quad L = \begin{Vmatrix} M & 0 \\ 0 & M \end{Vmatrix}, \quad D = \begin{Vmatrix} 0 & -A \\ A & 0 \end{Vmatrix} \\ P(t) = \begin{Vmatrix} P_x(t) & 0 \\ 0 & P_y(t) \end{Vmatrix}, \quad P(t+T) = P(t), \ T = 2\pi/\omega \end{gathered} \tag{3.6}$$

where obvious D is antisymmetric, and L and $P(t)$ are symmetric $2N \times 2N$ matrices.

Defining a $4N$-dimensional vector $\mathbf{Z} = (\mathbf{R}, L\mathbf{R}_t)^T$, we can write Eq.(3.6) in the following form (where I is the $2N \times 2N$ identity matrix):

$$\begin{Vmatrix} -D & -1 \\ I & 0 \end{Vmatrix} \mathbf{Z}_t = \begin{Vmatrix} P(t) & 0 \\ 0 & L^{-1} \end{Vmatrix} \mathbf{Z} \tag{3.7}$$

Eq.(3.7) is, by definition, canonical /7/, and therefore we can apply a theorem proved in /7/, according to which the vertices of the parametric resonance zones on the stability diagram, relative to the axes ω (the frequency of revolution) and ε (any parameter of the system), are the frequencies

$$\omega = (\omega_i + \omega_j)/n, \quad i, j, n = 1, 2, \ldots \tag{3.8}$$

where ω_i are the natural frequencies of the corresponding unperturbed system, i.e., the natural frequencies of the system with $b_i = c_i = 0$ in formulae (3.2). Therefore $\{\omega_i\} = \{p_{Mi}, p_i\}$, and the vertices of the parametric resonance zones are

$$\omega = 2p_{Mi}/n, \ 2p_i/n, \ (p_{Mi} + p_j)/n, \ (p_i + p_j)/n, \ (p_{Mi} + p_{Mj})/n \tag{3.9}$$

where p_{Mi} — unlike p_i — are the natural pendulum frequencies of the unperturbed (linearized) system with stiffness defined by formulae (2.5) (for p_i we have $C_x^k = C_y^k = C^k$).

Stability analysis in the case of rotary forced vibrations follows similar lines; the resulting vertices of the parametric resonance zones are

$$\omega = 2p_i/n, \quad (p_i + p_j)/n \tag{3.10}$$

4. For the fundamental resonance zones, i.e., $i = j$ in Eqs.(3.9) and (3.10), the regions of stability and instability may be determined using Floquet's theory /8/. Since the boundaries of these zones correspond to multiplier values /7/ $\rho = \exp(i\pi m) = \pm 1$, it follows that at the boundary of the dynamic instability region we have either a T-periodic ($\rho = 1$) or a T-antiperiodic ($\rho = -1$) non-trivial solution of Eq.(3.7), i.e., a solution

$$x^k = \sum_{n=0}^{\infty} (A_{xn}^k \cos n\omega t + B_{xn}^k \sin n\omega t) \tag{4.1}$$

$$x^k = \sum_{n=1}^{\infty} (E_{xn}^k \cos((2n-1)\omega t/2) + F_{xn}^k \sin((2n-1)\omega t/2)) \tag{4.2}$$

and similarly for y^k.

Substituting these expansions into the equations of motion (1.1), the matching conditions (1.2) and (2.6) and the boundary conditions (1.3), we apply the solution procedure using the method of initial parameters for all harmonics in expansions (4.1) and (4.2). Equating the coefficients of different harmonics, we obtain two infinite systems of linear equations with constant coefficients. The matrices of these systems are in block form:

$$\begin{Vmatrix} X_1^M & M_{12} & M_{13} & O & O & \ldots \\ M_{21} & X_2^M & M_{23} & M_{24} & O & \ldots \\ M_{31} & M_{31} & X_3^M & M_{34} & M_{35} & \ldots \\ O & M_{42} & M_{43} & X_4^M & M_{45} & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{Vmatrix} \tag{4.3}$$

$$\begin{Vmatrix} X^M_{1/2} & M_{12} & M_{13} & O & O & \dots \\ M_{21} & X^M_{3/2} & M_{23} & M_{24} & O & \dots \\ M_{31} & M_{32} & X^M_{5/2} & M_{34} & M_{35} & \dots \\ O & M_{42} & M_{43} & X^M_{7/2} & M_{45} & \dots \\ . & . & . & . & . & . \end{Vmatrix} \tag{4.4}$$

Here $X^M_{n/m}$ and M_{ij} are 8x8 matrices, and O is the 8x8 zero matrix. It is clear from (2.6) that the matrices M_{ij} are zero matrices if all $\delta^k = 0$.

It is obvious from the construction of the matrices (4.3) and (4.4) and from Eq.(1.6) that

$$X^M_{n/m}(\omega) = AA^M(\omega n/m)$$

and moreover the matrix $AA^M(\omega)$ is obtained from the matrix of the corresponding linear problem (formula (1.6)) by replacing the compliance of the bearings in which there are pendular vibrations in the horizontal direction with the reduced stiffnesses

$$C^k_M = C^k(1 - \delta^k/l_0{}^k) \tag{4.5}$$

Forming the determinants of the matrices (4.3) and (4.4) and equating them to zero, we obtain the conditions for the existence of non-trivial solutions (4.1), (4.2), and consequently also the equations of the curves bounding the unstable regions. As the number of terms in the expansions (4.1), (4.2) is increased, so does the order of the determinants of the matrices (4.3), (4.4). At the same time, the number of boundary curves determined is also increased and their positions in the stability diagram can be determined more accurately. Of course, if the damping is zero the determinants of the matrices AA^M will not vanish. In that case there is a loss of stability only at some fairly high level of the parameter δ and the non-zero matrices M_{ij}.

Thus, the parametric resonances in this non-linear system may be found explicitly by determining the natural frequencies of the corresponding linear systems.

5. To illustrate the method, let us consider the construction of boundary curves in the stability diagram in the case of a simple rotor represented by a homogeneous rod supported at its ends on a flexible bearing, hinged at $z = 0$, of stiffness C with clearance δ (at $z = L$), revolving at frequency ω.

The boundary conditions for the system are

$$x(0) = 0, \quad x_{zz}(0) = 0, \quad x_{zz}(L) = 0. \tag{5.1}$$

$$EJx_{zzz}(L) = C(1 - \delta/(x^2(L) + y^2(L))^{1/2})x(L) \tag{5.2}$$

(and analogous conditions for $y(z)$).

Consider the region of rotary vibrations. The solution in the first approximation is

$$x_0 = l_0(z) + r_1(z)\cos\omega t, \quad y_0 = r_1(z)\sin\omega t$$

where $l_0(z)$ is the static sag due to gravity and $r_1(z)$ the form of the vibrations due to the rotor unbalance.

For rotary vibrations we assume that

$$|r_1(L)| \gg l_0(L), \delta \tag{5.3}$$

Condition (5.2), linearized taking (5.3) into account and neglecting $\delta|r_1(L)$ compared with unity, becomes

$$EJx_{zzz}(L) = C(1 + \delta l_0(L)\cos\omega t/r_1^2(L))x(L) \tag{5.4}$$

In this case the equations for the functions x and y are separated, so that the orders of the blocks in the determinants of the matrices (4.3) and (4.4) are halved. We must consider the homogeneous Eqs.(1.1) of vibrations when the loads due to the gravitational and unbalance forces vanish

$$EJx_{zzzz} + \mu x_{tt} = 0 \tag{5.5}$$

and investigate the stability of the trivial solution of this equation, which satisfies the boundary conditions (5.1) and (5.4).

According to the previous arguments, the solutions at the boundaries of the regions of dynamic instability are

$$x = A_1(z)\sin\frac{\omega t}{2} + B_1(z)\cos\frac{\omega t}{2} + A_3(z)\sin\frac{3\omega t}{2} + \dots \tag{5.6}$$

$$x = B_0(z) + A_1(z)\sin\omega t + B_1(z)\cos\omega t + A_2(z)\sin 2\omega t + \dots \tag{5.7}$$

In the sequel we shall consider only curves defined by the expansion (5.6), retaining only the first two terms of the series. Substituting this expansion into Eq.(5.5), we obtain

$$A_1(z) = a_1 S_c^+(\rho z) + a_2 S_s^+(\rho z) + a_3 S_c^-(\rho z) + a_4 S_s^-(\rho z)$$

$$S_c^{\pm}(x) = (\mathrm{ch}\, x \pm \cos x)/2, \quad S_s^{\pm}(x) = (\mathrm{sh}\, x \pm \sin x)/2$$

$$\rho = (\mu\omega^2/(4EJ))^{1/4}$$

and an analogous expression for $B_1(z)$ (with a_i replaced by b_i), where a_i and b_i are arbitrary constants.

Substituting this result into the boundary conditions (5.1), (5.4), we find that $a_1 = a_3 = b_1 = b_3 = 0$. To determine the other arbirary constants, we equate the coefficients of $\sin(\omega t/2)$ and $\cos(\omega t/2)$, to obtain a system of fourth-order linear equations. This system separates into two independent second-order subsystems in the variables a_2, a_4 and b_2, b_4, respectively. The condition for a non-trivial solution to exist is that the determinant of one of these subsystems

$$I^-(\omega) I^+(\omega) = 0 \tag{5.8}$$

$$I^{\pm}(\omega) = \begin{Vmatrix} \rho^2 S_s^- & \rho^2 S_s^+ \\ p^{\pm} S_s^+ - \rho^3 S_c^- & p^{\pm} S_s^- - \rho^3 S_c^+ \end{Vmatrix}$$

$$p^{\pm} = C\,(1 \pm \delta l_0(L)/(2r_1^2(L)))/EJ$$

should vanish (the argument $z = \rho L$ in the functions $S_c^{\pm}, S_s^{\pm}$ is omitted).

When $\delta = 0$ this implies the usual frequency equation for a rotor in linear bearings. Its roots are obviously the numbers $\omega = 2p_i$ $(i = 1, 2, \ldots)$, where p_i are the natural frequencies of the corresponding linear systems. These are the parametric resonances in the stability diagram in the ω–δ plane.

Solving Eqs.(5.8) for δ, we mean the equations of the boundaries of the dynamic instability regions:

$$\delta(\omega) = \pm \frac{2r_1^2(L)}{l_0(L)} \left(\frac{\rho^3 EJ}{C} \frac{S_c^- S_s^+ - S_c^+ S_s^-}{(S_s^-)^2 - (S_s^+)^2} + 1 \right)$$

Putting $\rho L = q$, where q is the non-dimensional frequency of revolution $(\omega = 2q^2 L^{-2} (EJ/\mu)^{1/2})$, we obtain

$$\delta(q) = \pm r_1^2(L)/l_0(L) \quad (EJL^{-3}q^3(\mathrm{ctg}\, q - \mathrm{cth}\, q) + 2) \tag{5.9}$$

The approximate form of these curves is shown in Fig.2. The stable regions are hatched.

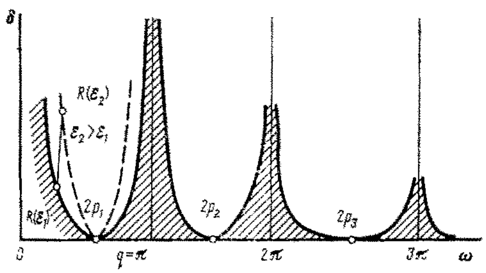

Fig.2

The following conclusions can be drawn.

As δ is increased, the unstable zones become wider.

Unstable regions corresponding to different parametric resonances become wider as the frequency of revolution increases.

If the parameter $\varepsilon = r_1^2/l_0$ is increased (i.e., an increase in the dynamic load or a drop in the static load), the ordinate of the point R on the curve $\delta(\omega)$ increases and the unstable zone for the region of rotary vibrations becomes narrower. It can be shown that in the region of pendular vibrations the pattern of variation in the width of the instability zone due to variation of ε is precisely the opposite.

Larger clearance causes a shift of the pendular resonances p_{Mi} toward lower frequencies of revolution (this is evident from formulae (4.5)) and a shift of the unstable regions corresponding to these resonances.

The unstable regions for parametric resonances $\omega = 2p_{Mi}/n$ and $\omega = 2p_i/n$ $(n > 1)$ can be obtained by retaining the necessary number of terms in Eqs.(5.7) and (5.8) and considering the appropriate type of motion (pendular or rotary vibrations).

REFERENCES

1. GRIGOR'YEV N.V., Motion of a loaded rotor having non-linear elements in the "rotor-stator" system, Tr. LKVVIA, 203, 1958.
2. GRIGOR'YEV N.V., On allowance for non-linear elements in calculation of vibrations in the "rotor-stator" system of a gas turbine engine. Tr. LKVVIA, 251, 1958.
3. MUSAYEV A.K., Effect of clearances in bearings on the critical velocities of revolving shafts. Tr. LKVVIA, 323, 1960.
4. BABAKOV I.M., Theory of Oscillations, Nauka, Moscow, 1965.
5. DIMENTBERG F.M. and KOLESNIKOV K.S. (Eds.), Vibrations in Engineering, 3, Mashinostroyeniye, Moscow, 1980.
6. KORN G.A. and KORN T.M., Mathematical Handbook for Scientists, and Engineers, McGraw-Hill, New York, 1968.
7. YAKUBOVICH V.N. and STARZHINSKII V.M., Parametric Resonance in Linear Systems, Nauka, Moscow, 1987.
8. MERKIN D.R., Introduction to the Theory of Stability of Motion, Nauka, Moscow, 1987.

Translated by D.L.

PMM U.S.S.R., Vol.54, No.1, pp.25-29, 1990
Printed in Great Britain

0021-8928/90 $10.00+0.00

FLOW OF A PLANE JET OF LIQUID FROM A RESERVOIR WITH FLEXIBLE WALLS NEAR A SCREEN*

V.P. ZHITNIKOV

The problem of the jet overhang created by a jet emerging through an orifice in a flexible barrier is considered. A numerical investigation is made of the mutual influence of the shape of the flexible reservoir walls and the jet parameters for different ratios of the pressure and distance to the screen.

The problem considered here is connected with calculations of the flow in flexible barriers of vessels on air cushions. Previous studies /1, 2/ have considered detached flow around a flexible casing near a screen, i.e., flow typical for the chamber scheme of formation of an air cushion. The study of flows in a jet scheme involves considerable computational complexity, and in this connection the problem is usually simplified by being split into two: computation of the shape of the casing on the assumption that the pressure distribution is a step function /3/, and computation of the jet flow from a nozzle device of given shape, in which context the nozzle is usually assumed to have straight walls /4, 5/. It is still not known to what degree the actual pressure distribution affects the shape of the casing, or how far the latter affects the jet parameters. The combined examination of both these problems in /6/, for the case in which the physical picture is symmetric about the vertical axis, shows that this influence, for real ratios of the width of the orifice in the casing to its length, is negligible. However, the problem when there is no symmetry remains open, in particular for large transverse pressure drops.

1. This appears is devoted to a numerical solution of the problem of a planar jet emerging from an orifice in a flexible barrier, in its exact non-linear steady-state formulation, for unequal pressures p_1 and p_0 and different casing lengths L_1 and L_2 from the edges of the orifice A and B to the attachment points A' and B' (Fig.1,a). The casing is assumed to be absolutely flexible (zero moment), weightless and inextensible; the liquid is assumed to be weightless, inviscid and incompressible. The casing is attached at its ends A' and B' to the vertical walls of the channel, and the ends A and B are assumed to be connected by a thin thread that does not obstruct the motion of the flow. The thread thereby keeps the ends of the

Prikl. Matem. Mekhan., 54, 1, 34-39, 1990